Operations Research
Anett Bekéné Rácz
1. Giapetto’s Woodcarving, Inc., manufactures two types of wooden toys: soldiers and trains. A soldier sells for $27 and uses $10 worth of raw materials. Each soldier that is manufactured increases Giapetto’s variable labor and overhead costs by $14. A train sells for $21 and uses $9 worth of raw materials. Each train built increases Giapetto’s variable labor and overhead costs by $10. The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wants to maximize weekly profit (revenues - costs). Formulate a mathematical model of Giapetto’s situation that can be used to maximize Giapetto’s weekly profit.

Solutions: Graphical, LinGo, Simplex

2. Farmer Joes must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labor per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labor per week. All wheat can be sold at $4 a bushel, and all corn can be sold at $3 a bushel. Seven acres of land and 40 hours per week of labor are available. Government regulations require that at least 30 bushels of corn be produced during the current year. Let $x_1$ number of acres of corn planted, and $x_2$ number of acres of wheat planted. Using these decision variables, formulate an LP whose solution will tell Farmer Jones how to maximize the total revenue from wheat and corn.

Solutions: Graphical, LinGo, Simplex

3. Truckco manufactures two types of trucks: 1 and 2. Each truck must go through the painting shop and assembly shop. If the painting shop were completely devoted to painting Type 1 trucks, then 800 per day could be painted; if the painting shop were completely devoted to painting Type 2 trucks, then 700 per day could be painted. If the assembly shop were completely devoted to assembling truck 1 engines, then 1,500 per day could be assembled; if the assembly shop were completely devoted to assembling truck 2 engines, then 1,200 per day could be assembled. Each Type 1 truck contributes $300 to profit; each Type 2 truck contributes $500. Formulate an LP that will maximize Truckco’s profit.

Solutions: Graphical, LinGo, Simplex

4. Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programs: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men. A 1-minute comedy ad costs $50,000, and a 1-minute football ad costs $100,000. Dorian would like the
commercials to be seen by at least 28 million high-income women and 24 million high-income men. Use linear programming to determine how Dorian Auto can meet its advertising requirements at minimum cost.

Solution: Graphical, LinGo, Simplex

5. Leary Chemical manufactures three chemicals: A, B, and C. These chemicals are produced via two production processes: 1 and 2. Running process 1 for an hour costs $4 and yields 3 units of A, 1 of B, and 1 of C. Running process 2 for an hour costs $1 and produces 1 unit of A and 1 of B. To meet customer demands, at least 10 units of A, 5 of B, and 3 of C must be produced daily. Graphically determine a daily production plan that minimizes the cost of meeting Leary Chemical’s daily demands.

Solution: Graphical, LinGo, Simplex

6. Furnco manufactures desks and chairs. Each desk uses 4 units of wood, and each chair uses 3. A desk contributes $40 to profit, and a chair contributes $25. Marketing restrictions require that the number of chairs produced be at least twice the number of desks produced. If 20 units of wood are available, formulate an LP to maximize Furnco’s profit. Then graphically solve the LP.

Solutions: Graphical, LinGo, Simplex

7. An auto company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, then 40 per day could be painted. If the paint shop were only painting cars, then 60 per day could be painted. If the body shop were only producing cars, then it could process 50 per day. If the body shop were only producing trucks, then it could process 50 per day. Each truck contributes $300 to profit, and each car contributes $200 to profit. Auto dealers require that the auto company produce at least 30 trucks and 20 cars. Use linear programming to determine a daily production schedule that will maximize the company’s profits.

Solutions: Graphical, LinGo, Simplex

8. My diet requires that all the food I eat come from one of the four “basic food groups” (chocolate cake, ice cream, soda, and cheesecake). At present, the following four foods are available for consumption: brownies, chocolate ice cream, cola, and pineapple cheesecake. Each brownie costs 50¢, each scoop of chocolate ice cream costs 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, I must ingest at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat. The nutritional content per unit of each food is shown in the following table. Formulate a linear programming model that can be used to satisfy my daily nutritional requirements at minimum cost.

<table>
<thead>
<tr>
<th></th>
<th>Calories</th>
<th>Chocolate (Ounces)</th>
<th>Sugar (Ounces)</th>
<th>Fat (Ounces)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownie</td>
<td>400</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>50</td>
</tr>
</tbody>
</table>
9. A little shop sales waffles and pancakes. They need two raw materials: flour and milk. A waffle requires 10 unit of flour and 13 unit of milk. A pancake requires 8 unit of flour and 6.5 unit of milk to produce. The shop has 800 unit of flour and 845 unit of milk available. (50 – waffle, 30 – pancake, profit maximization)

Solution: **Graphical, LinGo, Simplex**

10. Furnco manufactures tables and chairs. Each table and chair must be made entirely out of oak or entirely out of pine. A total of 150 board ft of oak and 210 board ft of pine are available. A table requires either 17 board ft of oak or 30 board ft of pine, and a chair requires either 5 board ft of oak or 13 board ft of pine. Each table can be sold for $40 and each chair for $15. Formulate an LP that can be used to maximize revenue.

Solution: **LinGo, Simplex**

**Duality and sensitivity analysis (With software output)**

Questions like in Ch. 5. p. 231.
Questions like in Ch. 5. p. 241. (Examples).
See the examples, right hand side ranges and dual prices pp. 232 – 237.

**Transportation problems (Manual transportation simplex solution, software solution)**

1. Powerco has three electric power plants that supply the needs of four cities.† Each power plant can supply the following numbers of kilowatt-hours (kwh) of electricity: plant 1—35 million; plant 2—50 million; plant 3—40 million (see Table below). The peak power demands in these cities, which occur at the same time (2 P.M.), are as follows (in kwh): city 1—45 million; city 2—20 million; city 3—30 million; city 4—30 million. The costs of sending 1 million kwh of electricity from plant to city depend on the distance the electricity must travel. Formulate an LP to minimize the cost of meeting each city’s peak power demand.

<table>
<thead>
<tr>
<th>From</th>
<th>To City 1</th>
<th>City 2</th>
<th>City 3</th>
<th>City 4</th>
<th>Supply (million kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant 1</td>
<td>8</td>
<td>6</td>
<td>10</td>
<td>9</td>
<td>35</td>
</tr>
<tr>
<td>Plant 2</td>
<td>9</td>
<td>12</td>
<td>13</td>
<td>7</td>
<td>50</td>
</tr>
<tr>
<td>Plant 3</td>
<td>14</td>
<td>9</td>
<td>16</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>Demand</td>
<td>45</td>
<td>20</td>
<td>30</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Solution: **LinGo, Northwest, Min. cost, Vogel**
2. Solve the following 3 supply point, 4 demand point transportation problem!

\[
\begin{array}{cccc}
C1 & C2 & C3 & C4 \\
W1 & 8 & 2 & 4 & 7 & 30 \\
W2 & 7 & 4 & 3 & 2 & 40 \\
W3 & 2 & 5 & 5 & 9 & 50 \\
\end{array}
\]

Solution: LinGo, Northwest, Min. cost, Vogel

3. Solve the following 4 supply points, 5 demand points transportation problem!

\[
\begin{array}{ccccc}
B1 & B2 & B3 & B4 & B5 \\
R1 & 8 & 7 & 3 & 4 & 2 & 12 \\
R2 & 6 & 2 & 7 & 5 & 10 & 17 \\
R3 & 7 & 5 & 3 & 3 & 1 & 25 \\
R4 & 4 & 9 & 9 & 8 & 2 & 35 \\
\end{array}
\]

Solution: LinGo, Northwest, Min. cost, Vogel

Assignment problems (Manual Hungarian method solution, software solution)

1. Doc Councillman is putting together a relay team for the 400-meter relay. Each swimmer must swim 100 meters of breaststroke, backstroke, butterfly, or freestyle. Doc believes that each swimmer will attain the times given in the following table. To minimize the team’s time for the race, which swimmer should swim which stroke?

<table>
<thead>
<tr>
<th>Swimmer</th>
<th>Free</th>
<th>Breast</th>
<th>Fly</th>
<th>Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gary Hall</td>
<td>54</td>
<td>54</td>
<td>51</td>
<td>53</td>
</tr>
<tr>
<td>Mark Spitz</td>
<td>51</td>
<td>57</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>Jim Montgomery</td>
<td>50</td>
<td>53</td>
<td>54</td>
<td>56</td>
</tr>
<tr>
<td>Chet Jastremski</td>
<td>56</td>
<td>54</td>
<td>55</td>
<td>53</td>
</tr>
</tbody>
</table>
2. Machineco has five machines and five jobs to be completed. Each machine must be assigned to complete one job. The time required to set up each machine for completing each job is shown in the following table. Machineco wants to minimize the total setup time needed to complete the four jobs. Use linear programming to solve this problem!

<table>
<thead>
<tr>
<th>Machines</th>
<th>JOBS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J1</td>
</tr>
<tr>
<td>M1</td>
<td>2</td>
</tr>
<tr>
<td>M2</td>
<td>4</td>
</tr>
<tr>
<td>M3</td>
<td>6</td>
</tr>
<tr>
<td>M4</td>
<td>4</td>
</tr>
<tr>
<td>M5</td>
<td>9</td>
</tr>
</tbody>
</table>

Solution: Hungarian method

3. Solve the following 3 machines 3 jobs assignment problem (minimize the total cost in the following matrix)!

<table>
<thead>
<tr>
<th></th>
<th>J1</th>
<th>J2</th>
<th>J3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>10</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>M2</td>
<td>15</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>M3</td>
<td>7</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

Solution: Hungarian method

Theoretical questions:

1. Definition of Linear Programming Problem (LP) – Ch 3, p. 53.
2. Definition of Feasible region (feasible set) – Ch 3, p. 54.
4. Definition of standard form – Ch 4, p. 127.
5. Definitions of basic or nonbasic variables – Ch 4, p. 130.
7. Definition of shadow price – Ch 5, p. 230. also p. 252.
9. Balancing a transportation problem – Ch. 7. p. 364.-
10. Solving transportation problem on the Computer (LinGo) – Ch. 7. p. 369. (you can use lingo sample problem TRAN.lng or create your own model)
11. Steps of the transportation simplex algorithm – Ch. 7. p. 388.

Remarks:

- Know all the abbreviations in the theorems and definitions.
- You can find useful help in the Winston book for practical part also.
- Exam contains two part: practical part (4 exercise) (at least 50% of this part for the signature) and theoretical part (3 theoretical questions) all from these above.